Spin squeezing and concurrence

Xiaolei Yin,[1] Xiaoqian Wang,[1, 2] Jian Ma,[1] and Xiaoguang Wang[1, *]

[1]*Zhejiang Institute of Modern Physics, Department of Physics, Zhejiang University, Hangzhou 310027, China*

[2]*Department of Physics, Changchun University of Science and Technology, Changchun 130022, China*

We study the relations between spin squeezing and concurrence, and find that they are qualitatively equivalent for an ensemble of spin-1/2 particles with exchange symmetry and parity, if we adopt the spin squeezing criterion given by the recent work (G. Tóth *et al.* Phys. Rev. Lett. **99**, 250405 (2007)). This suggests that the spin squeezing has more intimate relations with pairwise entanglement other than multipartite entanglement. We exemplify the result by considering a superposition of two Dicke states.

PACS numbers: 03.65.Ud,03.67.2a

I. INTRODUCTION

As an important resource of quantum information and computation, entanglement [1, 2] has attracted much attention in recent years [3–12]. How to measure and detect entanglement is crucial for both theoretical investigations and potential practical applications [13, 14]. The entanglement of a two-qubit system can be well quantified by the concurrence [15, 16]. However, quantification of many-body entanglement is still an open question in quantum information.

It is well known that there are close relations between entanglement and spin squeezing [17–24]. There are several definitions of spin squeezing parameters [18–20], which are studied in different papers. The squeezing parameter ξ_R^2 defined by Wineland *et al.* is closely related to multipartite entanglement. It has been proven that [18], for an ensemble of spin-1/2 particles, if this squeezing parameter is less than one, the state is entangled. The advantages of spin squeezing parameters in detecting entanglement have been shown in both theoretical and experimental aspects.

The squeezing parameter ξ_S^2 defined by Kitagawa and Ueda is relevant to pairwise entanglement [21]. And for states with exchange symmetry and parity, a simple quantitative relation between ξ_S^2 and concurrence was given [22]. Furthermore, it has been shown that the spin squeezing and pairwise entanglement are equivalent for states generated by the one-axis twisting Hamiltonian [22]. However, even for states with a fixed parity, such as the states generated by one-axis twisting Hamiltonian with a transverse field, ξ_S^2 is not always equivalent to concurrence [25]. Inspired by recent works [26, 27], where a set of generalized spin squeezing inequalities are developed, one can define another spin squeezing parameter ξ_T^2 from one of the inequalities [28]. Similar to parameter ξ_R^2, one advantage of the parameter ξ_T^2 is that we can firmly say that the state is entangled if $\xi_T^2 < 1$. However, if parameter $\xi_S^2 < 1$, we cannot say the state is entangled, although this parameter is closely related to entanglement.

Reference [21] found that spin squeezing according to parameter ξ_S^2 is equivalent to the minimal pairwise correlation $\mathcal{C}_{\vec{n}_\perp,\vec{n}_\perp}$ along the direction $\vec{n}_\perp$ (which is perpendicular to the mean spin direction) for symmetric states. It was further found [29] that for the symmetric states, the spin squeezing defined via ξ_T^2 is equivalent to minimal pairwise correlation $\mathcal{C}_{\vec{n},\vec{n}}$ along an arbitrary direction $\vec{n}$. For states with a fixed parity, the relations between the two parameters ξ_S^2 and ξ_T^2 are more evident. It will be seen from Sec. 3 that ξ_T^2 contains the term ξ_S^2, and the spin squeezing results from just the competition between pairwise correlation along the direction $\vec{n}_\perp$ and that along the mean spin direction. So, in this sense, the parameter ξ_T^2 is a natural generalization of ξ_S^2.

We find that for states with exchange symmetry and parity, the spin squeezing parameter ξ_T^2 is qualitatively equivalent to the concurrence in characterizing pairwise entanglement. In other words, the spin squeezing parameter and concurrence emerge and vanish at the same time. This finding is of significance to entanglement detection in experiments. As we all know, entanglement detectors such as entropy and concurrence are generally not easy to measure, especially for physical systems like BEC, for which we cannot address individual atoms. However, spin squeezing parameters are relatively easy to measure in experiments, since they only consist of expectations and variances of collective angular momentum operators. Nevertheless, the traditional spin squeezing parameter ξ_S^2 is not always equivalent to concurrence even for states with exchange symmetry and parity. As ξ_T^2 is qualitatively equivalent to concurrence for an ensemble of spin-1/2 particles with exchange symmetry and parity, it is better than ξ_S^2 in detecting pairwise entanglement.

The paper is organized as follows: In Sec. II, we give the definitions of the two spin squeezing parameters and concurrence. In Sec. III, we give the concrete forms of the spin squeezing parameters and the concurrence for states with exchange symmetry and parity. The relations between these two parameters and concurrence were given in Sec. IV. We exemplify the result by considering superpositions of Dicke states in Sec. V. Finally, Sec. VI is

*Electronic address: xgwang@zimp.zju.edu.cn

devoted to conclusion.

II. SPIN SQUEEZING PARAMETERS AND CONCURRENCE

To study spin squeezing, we consider an ensemble of N spin-1/2 particles. For the sake of describing many-particle systems, we use the total angular momentum operators

$$J_\alpha = \frac{1}{2}\sum_{k=1}^{N} \sigma_{k\alpha}, \quad (\alpha = x, y, z), \tag{1}$$

where $\sigma_{k\alpha}$ are the Pauli matrices for the k-th spin. Now, we give the definitions of the two spin squeezing parameters. The first one is defined as [19],

$$\xi_S^2 = \frac{4 \min(\Delta J_{\vec{n}_\perp})^2}{N}, \tag{2}$$

where $\vec{n}_\perp$ is perpendicular to the mean spin direction $\vec{n} = \frac{\langle \vec{J} \rangle}{|\langle \vec{J} \rangle|}$. Since the system has the exchange symmetry, the total angular momentum is $j = \frac{N}{2}$. For spin coherent states [19], $\Delta J_{\vec{n}_\perp} = \frac{j}{2}$, and $\xi_S^2 = 1$. In the following, we consider states with exchange symmetry.

The next spin squeezing parameter is based on the generalized spin squeezing inequalities given by Tóth *et al.* [27], and is defined as [28]

$$\xi_T^2 = \frac{\lambda_{\min}}{\langle \vec{J}^2 \rangle - \frac{N}{2}}, \tag{3}$$

where $\lambda_{\min}$ is the minimum eigenvalue of

$$\Gamma = (N-1)\gamma + G \tag{4}$$

with $G_{kl} = \frac{1}{2}\langle J_k J_l + J_l J_k \rangle$, $(k, l = x, y, z)$ the correlation matrix, and $\gamma_{kl} = G_{kl} - \langle J_k \rangle \langle J_l \rangle$ the covariance matrix. For our states, $\langle \vec{J}^2 \rangle - \frac{N}{2} = j(j+1) - j = j^2$.

The two-qubit entanglement is quantified by the concurrence, whose definition is given by [16]

$$C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}, \tag{5}$$

where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ are the square roots of eigenvalues of $\tilde{\rho}\rho$. Here ρ is the reduced density matrix of the system, and

$$\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y), \tag{6}$$

where ρ^* is the conjugate of ρ. If $C > 0$, the system displays pairwise entanglement.

III. STATES WITH PARITY

To study the relations between spin squeezing parameters and concurrence, we consider a class of states with even (odd) parity, which means a state in the $(2j+1)$-dimensional space with only even (odd) excitations of spins. These kinds of states are widely studied, e.g., the states generated by the one-axis twisting model [19]. The states with even parity possess important properties, $\langle J_\alpha \rangle = 0$, $\langle J_\alpha J_z \rangle = \langle J_z J_\alpha \rangle = 0$, $\alpha = x, y$, which means the mean spin direction is along the z-axis, and the covariances between J_z and J_α are zero. Thus, equation (4) becomes

$$\Gamma = \begin{pmatrix} N\left\langle J_x^2 \right\rangle & \frac{N\left\langle [J_x, J_y]_+ \right\rangle}{2} & 0 \\ \frac{N\left\langle [J_x, J_y]_+ \right\rangle}{2} & N\left\langle J_y^2 \right\rangle & 0 \\ 0 & 0 & N(\Delta J_z)^2 + \langle J_z \rangle^2 \end{pmatrix}, \tag{7}$$

where $[A, B]_+ = AB + BA$, and equation (3) reduces to [28]

$$\xi_T^2 = \min\left\{\xi_S^2, \varsigma^2\right\}, \tag{8}$$

where

$$\begin{aligned} \varsigma^2 &= \frac{4}{N^2}\left[N(\Delta J_z)^2 + \langle J_z \rangle^2\right] \\ &= 1 + (N-1)\left(\langle \sigma_{1z}\sigma_{2z} \rangle - \langle \sigma_{1z} \rangle^2\right) \\ &= 1 + (N-1)C_{zz}, \end{aligned} \tag{9}$$

with C_{zz} the two-spin correlation function along z direction. The explicit form of ξ_S^2 could be obtained as [22]

$$\begin{aligned} \xi_S^2 &= \frac{2}{N}\left(\langle J_x^2 + J_y^2 \rangle - |\langle J_-^2 \rangle|\right) \\ &= 1 - 2(N-1) \\ &\quad \times \left[|\langle \sigma_{1-}\sigma_{2-} \rangle| - \frac{1}{4}\left(1 - \langle \sigma_{1z}\sigma_{2z} \rangle\right)\right], \end{aligned} \tag{10}$$

where we have used the following relations

$$\begin{aligned} \langle J_\alpha \rangle &= \frac{N}{2}\langle \sigma_{1\alpha} \rangle, \\ \langle J_\alpha^2 \rangle &= \frac{N}{4} + \frac{N(N-1)}{4}\langle \sigma_{1\alpha}\sigma_{2\alpha} \rangle, \\ \langle J_-^2 \rangle &= N(N-1)\langle \sigma_{1-}\sigma_{2-} \rangle, \end{aligned} \tag{11}$$

which connect the local expectations with collective ones.

For such states, the significance of ξ_S^2 and ξ_T^2 and the relations between them are clear. According to the parameter ξ_S^2, a state is squeezed when the minimum variance of angular momentum in the $\vec{n}_\perp$-plane is smaller than $\frac{j}{2}$, while according to ξ_T^2, the variance in the mean spin direction $\vec{n}$ is also considered. Equation (9) represents the pairwise correlation along the mean spin direction, and this can be viewed as a complement of ξ_S^2, which only considers squeezing in the $\vec{n}_\perp$-plane. Thus, ξ_T^2 can be regarded as a generalization of ξ_S^2, and when $\xi_S^2 < \varsigma^2$, the parameter ξ_T^2 reduces to ξ_S^2.

To calculate concurrence, we first need to calculate the two-body reduced density matrix, which can be written

as [22]

$$\rho = \begin{pmatrix} v_+ & 0 & 0 & u^* \\ 0 & y & y & 0 \\ 0 & y & y & 0 \\ u & 0 & 0 & v_- \end{pmatrix}, \tag{12}$$

where

$$\begin{aligned} v_\pm &= \frac{1}{4}\left(1 \pm 2\langle\sigma_{1z}\rangle + \langle\sigma_{1z}\sigma_{2z}\rangle\right), \\ u &= \langle\sigma_{1-}\sigma_{2-}\rangle, \quad y = \frac{1}{4}\left(1 - \langle\sigma_{1z}\sigma_{2z}\rangle\right), \end{aligned} \tag{13}$$

in the basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. Then the concurrence is given by

$$C = 2\max\left\{0,\ |u| - y,\ y - \sqrt{v_+v_-}\right\}. \tag{14}$$

One key observation is that

$$y^2 - v_+v_- = -\frac{1}{4}C_{zz}. \tag{15}$$

Thus,

$$\varsigma^2 = 1 - 4(N-1)(y + \sqrt{v_+v_-})(y - \sqrt{v_+v_-}). \tag{16}$$

From equations (9), (10), and (13), we obtain

$$\begin{aligned} \xi_S^2 &= 1 - 2(N-1)\left(|u| - y\right), \\ \xi_T^2 &= \min\{1 - 2(N-1)\left(|u| - y\right), \\ &\quad 1 - 4(N-1)(y + \sqrt{v_+v_-})(y - \sqrt{v_+v_-})\}. \end{aligned} \tag{17}$$

Now, one can see that the squeezing parameters are related to the concurrence shown in equation (14). The relations between ξ_S^2 and C have been studied [22]. In the following, we consider the squeezing parameter ξ_T^2, and prove that it is qualitatively equivalent to the concurrence in detecting pairwise entanglement.

IV. RELATIONS BETWEEN SPIN SQUEEZING PARAMETERS AND CONCURRENCE

Firstly, we prove that for a state with exchange symmetry and parity, if concurrence $C > 0$, it must be spin squeezed according to the criterion $\xi_T^2 < 1$. From equation (14) we note that when $C > 0$, there are two cases, $C = |u| - y > 0$ or $C = y - \sqrt{v_+v_-} > 0$. However, since the density matrix ρ is positive, we find $\sqrt{v_+v_-} \geq |u|$, then immediately

$$\left(|u| - y\right)\left(y - \sqrt{v_+v_-}\right) \leq 0, \tag{18}$$

which means $|u| - y$ and $y - \sqrt{v_+v_-}$ cannot be positive simultaneously. Therefore, if $C > 0$, we have [30]

$$C = \begin{cases} 2\left(|u| - y\right), & |u| > y, \\ 2\left(y - \sqrt{v_+v_-}\right), & y > \sqrt{v_+v_-}. \end{cases} \tag{19}$$

According to equations (8) and (17), we get the following relations

$$\xi_T^2 = \begin{cases} 1 - (N-1)\,C, & |u| > y, \\ 1 - 2\,(N-1)\left(y + \sqrt{v_+v_-}\right)C, & y > \sqrt{v_+v_-}, \end{cases} \tag{20}$$

since $C > 0$, there always be $\xi_T^2 < 1$.

Now, we prove that if the state is spin squeezed $\left(\xi_T^2 < 1\right)$, concurrence $C > 0$. If $\xi_T^2 < 1$, there are two cases, $\xi_T^2 = \xi_S^2 < 1$ or $\xi_T^2 = \varsigma^2 < 1$. As discussed above, according to equations (17) and (18), $\xi_S^2 < 1$ and $\varsigma^2 < 1$ could not occur simultaneously. Therefore, if $\xi_T^2 = \xi_S^2 < 1$, we have [31]

$$C = \frac{1 - \xi_T^2}{N - 1}, \tag{21}$$

while if $\xi_T^2 = \varsigma^2 < 1$, we have

$$C = \frac{1 - \xi_T^2}{2\,(N-1)\left(y + \sqrt{v_+v_-}\right)}. \tag{22}$$

Therefore, if the state is squeezed, concurrence $C > 0$.

The relations between spin squeezing and concurrence is displayed in Table I, and we can see that, for a symmetric state, $\xi_T^2 < 1$ is qualitatively equivalent to $C > 0$, that means spin squeezing according to ξ_T^2 is equivalent to pairwise entanglement. Although $\xi_S^2 < 1$ indicates $C > 0$, when $C = 2(y - \sqrt{v_+v_-}) > 0$, we find $\xi_S^2 > 1$. Therefore, a spin-squeezed state ($\xi_S^2 < 1$) is pairwise entangled, while a pairwise entangled state may not be spin-squeezed according to the squeezing parameter ξ_S^2. Then, we come to the conclusion that for states with exchange symmetry and parity, the spin squeezing parameter ξ_T^2 is qualitatively equivalent to the concurrence in characterizing pairwise entanglement. In the following, we will give some examples and applications of our result.

V. EXAMPLES AND APPLICATIONS

We first consider a superposition of Dicke states with parity, and then consider states without a fixed parity. The states under consideration are all based on Dicke states [32], and are defined as

$$|n\rangle_N \equiv |\frac{N}{2}, -\frac{N}{2} + n\rangle, \quad n = 0, \ldots, N, \tag{23}$$

where $|0\rangle_N \equiv |\frac{N}{2}, -\frac{N}{2}\rangle$ denotes a state for which all spins are in the ground states, and n is the excitation number of spins. Such states are elementary in atomic physics, and may be conditionally prepared in experiments with quantum non-demolition technique [33–35].

As we consider the state with even parity, we choose a simple superposition of Dicke states as

$$|\psi_D\rangle = \cos\theta|n\rangle_N + e^{i\varphi}\sin\theta|n+2\rangle_N, \quad n = 0, \ldots, N-2 \tag{24}$$

TABLE I: Spin squeezing parameters and concurrence for states with exchange symmetry and parity.

	Pairwise entangled ($C>0$)		Unentangled
Concurrence	$C=2(\lvert u\rvert-y)>0$	$C=2(y-\sqrt{v_+v_-})>0$	$C=0$
ξ_S^2	$\xi_S^2=1-(N-1)C<1$	$\xi_S^2>1$	$\xi_S^2\geq 1$
ξ_T^2	$\xi_T^2=1-(N-1)\,C<1$	$\xi_T^2=1-2(N-1)(y+\sqrt{v_+v_-})\times C<1$	$\xi_T^2\geq 1$

with the angle $\theta\in[0,\pi)$ and the relative phase $\varphi\in[0,2\pi)$. We can easily check that, for the superposition state in equation (24), the mean spin direction is along the z-axis. The expressions for the relevant spin expectation values can be obtained as

$$\begin{aligned}\langle J_z\rangle &= m+2\sin^2\theta,\\ \langle J_z^2\rangle &= m^2+(4m+1)\sin^2\theta,\\ \langle J_+^2\rangle &= \langle J_-^2\rangle=\frac{1}{2}e^{i\varphi}\sin 2\theta\sqrt{\mu_n},\end{aligned}\tag{25}$$

where $m=n-\frac{N}{2}$, and $\mu_n=(n+1)\,(n+2)\,(N-n)\,(N-n-1)$.

By substituting equations (25) to equation (9) and (10), it is easy to get

$$\begin{aligned}\xi_S^2 &= 1-\frac{2}{N}\{\lvert\sin\theta\cos\theta\rvert\sqrt{\mu_n}\\ &\quad -4[m^2+4(m+1)\sin^2\theta]-N^2\}\end{aligned}\tag{26}$$

and

$$\begin{aligned}\varsigma^2 &= \frac{4}{N}\left[m^2+4\,(m+1)\sin^2\theta\right]\\ &\quad -\frac{4(N-1)}{N^2}\left[m+2\sin^2\theta\right]^2.\end{aligned}\tag{27}$$

From the results in [36] we can easily get [30]

$$\begin{aligned}u &= \frac{e^{i\varphi}\sin 2\theta}{2N(N-1)}\sqrt{\mu_n},\\ y &= \frac{N}{4(N-1)}-\frac{[m^2+4(m+1)\sin^2\theta]}{N(N-1)},\\ \sqrt{v_+v_-} &= \frac{\sqrt{(N^2-2N+4\,\langle J_z^2\rangle)^2-16(N-1)^2\,\langle J_z\rangle^2}}{4N(N-1)}.\end{aligned}\tag{28}$$

Insert equation (28) to equation (14), one can get the expression of concurrence.

In figure 1, we plot these two spin squeezing parameters and concurrence versus θ in one period. We observe that for $\theta\in(0,\pi/3)\cup(2\pi/3,\pi)$, $\xi_T^2=\xi_S^2<1$, therefore the state is spin squeezed in the x-y plane, moreover, as $C>0$, the state is pairwise entangled. For $\theta\in(\pi/3,2\pi/3)$, it is obviously that the state is also pairwise entangled, since $C>0$, while spin squeezing occurs in the z-axis since $\xi_T^2<1$ while $\xi_S^2>1$. The results show clearly that $\xi_T^2<1$ is equivalent to $C>0$. But if we adopt $\xi_S^2<1$ as squeezing parameter, the spin squeezing is not qualitatively equivalent to concurrence.

The equivalence of $\xi_T^2<1$ and $C>0$ for states with

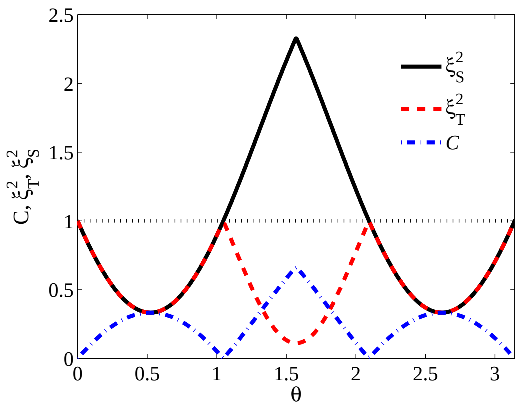

FIG. 1: Spin squeezing parameters ξ_S^2 and ξ_T^2, and concurrence as functions of θ for $N=3$ and $n=0$.

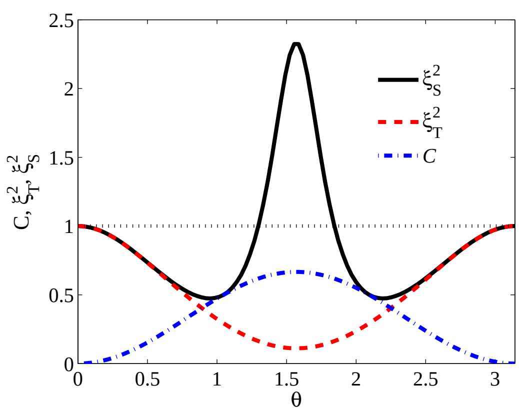

FIG. 2: Spin squeezing parameters ξ_S^2 and ξ_T^2, and concurrence as functions of θ for $N=3$ and $n=0$. The numerical result give $\xi_S^2=7/3$, $\xi_T^2=1/9$, and $C=2/3$, when $\theta=\pi/2$.

parity has been demonstrated above. Here, we discuss states without parity to see the relations between spin squeezing and entanglement. For simplicity, we choose

$$|\psi_D\rangle = \cos\theta|n\rangle_N + e^{i\varphi}\sin\theta|n+1\rangle_N, \quad n = 0,\ldots,N-1. \tag{29}$$

Specifically, if $\theta = \frac{\pi}{2}$, $n = 0$ or $n = N-2$, the above state degenerates to the W state. Moreover, when $N = 3$, equation (29) reduces to

$$|\psi_D\rangle = \frac{1}{\sqrt{3}}(|110\rangle + |101\rangle + |011\rangle). \tag{30}$$

The two-qubit reduced density matrix becomes

$$\rho = \frac{1}{3}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{31}$$

and using equation (14) we find $C = \frac{2}{3}$. We can also get the expectations of spin components, $\langle J_z\rangle = -\frac{1}{2}$, $\langle J_x^2\rangle = \langle J_y^2\rangle = \frac{7}{4}$, $\langle J_y^2\rangle = \frac{1}{4}$, and then we can easily get the spin squeezing parameters, $\xi_S^2 = \frac{7}{3}$ and $\xi_T^2 = \frac{1}{9}$. The numerical results for ξ_T^2 is displayed in figure 2, which coincide with the special result. It is interesting to see that, although $|\psi_D\rangle$ has no parity, the state is entangled ($C > 0$) and is spin squeezed according to ξ_T^2 in the entire interval. However, according to parameter ξ_S^2 the state is not squeezed in the middle region. Therefore, we find that ξ_T^2 is more effective than ξ_S^2 in detecting pairwise entanglement.

VI. CONCLUSION

In conclusion, we have studied the relations between spin squeezing and pairwise entanglement. We have considered two types of spin squeezing parameters ξ_S^2 *and* ξ_T^2 , and the pairwise entanglement is characterized by concurrence C. We find that, for states with exchange symmetry and parity, spin squeezing according to ξ_T^2 is qualitatively equivalent to pairwise entanglement. In detecting pairwise entanglement, parameter ξ_T^2 is more effective than parameter ξ_S^2.

It is important to emphasize that, the above conclusion can be extended to the states without (even or odd) parity. For states with properties $\langle J_\alpha\rangle = 0$, $\langle J_\alpha J_z\rangle = \langle J_z J_\alpha\rangle = 0$, $\alpha = x, y$, we can have the same conclusion that spin squeezing and pairwise entanglement are qualitatively equivalent. The following superposition of Dicke states are examples: $|\psi_{D'}\rangle = \cos\theta|n\rangle_N + e^{i\varphi}\sin\theta|n+n'\rangle_N$, $n = 0,\ldots,N-n'$, for all $n' \geq 3$. As we have seen, parameter ξ_S^2 is a key factor in ξ_T^2 for our states. The present results imply that the spin squeezing has more intimate relations with pairwise entanglement.

Acknowledgements

This work is supported by NSFC with grant No. 10874151 and 10935010; and the Fundamental Research Funds for the Central Universities.

[1] A. Einstein, B. Podolsky, and N. Rosen. Phys. Rev. **47**, 777 (1935).
[2] E. Schrödinger, Naturwissenschaften **23**, 807 (1935).
[3] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[4] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
[5] A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).
[6] X. Wang, Phys. Rev. A **64**, 012313 (2001).
[7] X. Wang, H. Fu, and A.I. Solomon, J. Phys. A: Math. Gen. **34**, 11307 (2001).
[8] J. W. Pan, M. Daniell, S. Gasparoni, G. Weihs, and A. Zeilinger, Phys. Rev. Lett. **86**, 4435 (2001).
[9] G. Vidal, J.I. Latorre, E. Rico, and A. Kitaev, Phys. Rev. Lett. **90**, 227902 (2003).
[10] J. Vidal, G. Palacios, and R. Mosseri, Phys. Rev. A **69**, 022107 (2004).
[11] D. Leibfried, M. D. Barrett, T. Schaetz, J. Chiaverini, W. M. Itano, J. D. Jost, C. Langer, and D. J. Wineland, Science **304**, 1476 (2004).
[12] A. André, A. S. Sørensen. and M. D. Lukin, Phys. Rev. Lett. **92,** 230801 (2004).
[13] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, Phys. Rev. A, **54**, 3824 (1996).
[14] V. Vedral, M. B. Plenio, M. A. Rippin, and P. L. Knight, Phys. Rev. Lett. **78**, 2275 (1997).
[15] S. Hills and W. K. Wootters, Phys. Rev. Lett. **78**, 5022 (1997).
[16] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).
[17] A. Sørensen, K. Mølmer, Phy. Rev. Lett. **83**, 2274 (1999).
[18] A. Sørensen, L. M. Duan, J. I. Cirac, and P. Zoller, Nature (London) **409**, 63 (2001).
[19] M. Kitagawa and M. Ueda, Phys. Rev. A, **47**, 5138 (1993).
[20] D. J. Winland, J. J. Bollinger, W. M. Itano, Phys. Rev. A, **50**, 67 (1994).
[21] D. Ulam-Orgikh and M. Kitagawa, Phys. Rev. A, **64**, 052106 (2001).
[22] X. Wang and B. C. Sanders, Phys. Rev. A, **68**, 012101 (2003).
[23] G. R. Jin and S. W. Kim, Phys. Rev. A, **76**, 043621 (2007).
[24] M. Jafarpour, A. Akhoundm, Phy. Lett. A **372**, 2374 (2008).
[25] X. Wang, Phys. Lett. A, **331**, 164 (2004).
[26] G. Tóth, C. Knapp, O. Gühne, and H. J. Briegel, Phys. Rev. Lett. **99**, 250405 (2007).

[27] G. Tóth, C. Knapp, O. Gühne, and H. J. Briegel, Phys. Rev. A **79**, 042334 (2009).
[28] X. Wang, A. Miranowicz, Y. Liu, C. Sun, and F. Nori, Phys. Rev. A **81**, 022106 (2010).
[29] X. Wang, J. Ma, L. Song, X. Zhang, and X. Wang, e-print arXiv: 1010.0109v1.
[30] J. Vidal, Phys. Rev. A **73**, 062318 (2006).
[31] J. Vidal, G. Palacios, and C. Aslangul, Phy. Rev. A **70**, 062304 (2004).
[32] R. H. Dicke, Phys. Rev. **93**, 99 (1954).
[33] K. Mølmer, Eur. Phys. J. D **5**, 301 (1998).
[34] A. Kuzmich, N. P. Bigelow, and L. Mandel, Europhys. Lett. **42**, 481 (1998).
[35] K. Lemr and J. Fiurášek, Phys. Rev. A **79**, 043808 (2009).
[36] X. Wang and K. Mølmer, Eur. Phys. J. D **18**, 385 (2002).